**Registration no:** | | | | | | | | | | |

**Total Number of Pages: 03**

**MCA**
**MCC103**

## 1st Semester Back Examination 2016-17
## DISCRETE MATHEMATICS
### BRANCH: MCA
### Time: 3 Hours
### Max Marks: 70
### Q.CODE: Y626

**Answer Question No.1 which is compulsory and any five from the rest.**

**The figures in the right hand margin indicate marks.**

**Q1 Answer the following questions:** **(2 x 10)**

**a)** If p is true and q is false, find the truth value of the following
$\neg(p \wedge q) \vee \neg(q \Leftrightarrow p)$

**b)** Determine the truth value of $\forall x \exists y (x^2 = y)$, if the domain of each variable consists of all real numbers.

**c)** What rule of inference is used in the argument?
If it snows today, the university will close. The university is not close today.
Therefore, it did not snow today.

**d)** Prove that if n is an integer and 3n+2 is odd, then n is odd.

**e)** A sequence is defined by the recurrence relation $a_{n+1} = 3a_n + 1$ with $a_0 = 10$. Then find the value of $a_1 + a_2 + a_3$.

**f)** How many edges are there in a graph with 10 vertices each of degree 6?

**g)** What is a Hamiltonian Graph? Gibe an example of a Hamiltonian Graph.

**h)** Show that in a Boolean algebra, the complement of an element is unique.

**i)** Find the inverse of $-i$ in the multiplicative group, $\{1, -1, i, -i\}$.

**j)** Consider the encoding function $e: B^2 \rightarrow B^5$ defined by

e (00)=00000 e(10)=10110
e(01)=01101 e(11)=11011

Find the minimum distance.

**Q2 a)** Let $m$ and $n$ be integers. Prove that $n^2 = m^2$ if and only if $n$ is $m$ or $n$ is $-m$. **(5)**

**b)** Show by mathematical induction that $(11)^{n+2} + (12)^{2n+1}$ is divisible by 133. **(5)**

**Q3 a)** Explain the principle of inclusion exclusion. Using this principle, find out the number of solutions for the equation $x_1 + x_2 + x_3 = 13$, where $x_1, x_2, x_3$ are non-negative integers less than six. **(5)**

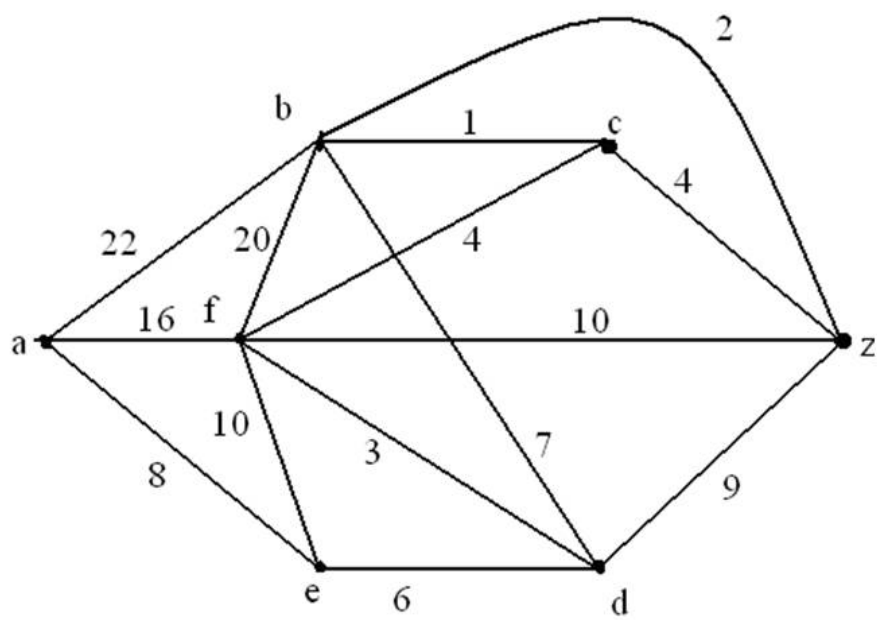
**b)** Using Warshall's algorithm find the transitive closure of the relation whose matrix representation is given by **(5)**

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

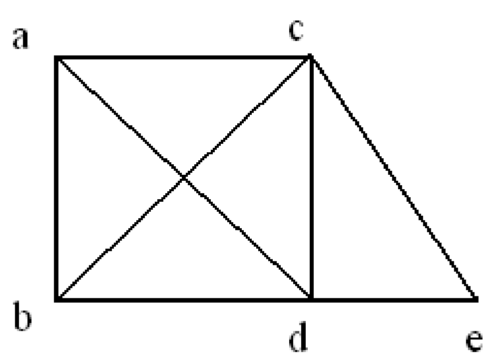
**Q4 a)** Prove that for R to be one equivalence relation on a set A, the following statements are equivalent: (i) a R b (ii) [a] = [b] (iii) $[a] \cap [b] = \phi$ . **(5)**

**b)** Determine the discrete numeric function corresponding to the generating function $A(z) = \dfrac{1}{z^2 - 5z + 6}$ **(5)**

**Q5 a)** Using Dijkstra's algorithm find the shortest path from a to z. **(5)**




**b)** How many path of length three and four are there from a to d in the simple graph given below? **(5)**




**Q6 a)** Let R be a symmetric relation on a set A. Then show that the following statements are equivalent. **(5)**

(a) R is an undirected tree.
(b) R is connected and acyclic.

**b)** What do you mean by binary tree? Show that the maximum number of vertices in a binary tree of height h is $2^{h+1}-1$ **(5)**

**Q7 a)** Consider the partial order $\le$ defined on A as follows: m$\le$n iff m divides n, where A = {2, 4, 6, 9, 12, 18, 36, 48, 60, 72}. Draw the Hasse diagram and answer the following questions: **(5)**

- **(i)** Is there a greatest element?
- **(ii)** Is there a least element?
- **(iii)** List the minimal elements.
- **(iv)** Find the least upper bound of (4, 9).

**b)** Prove that any linear order is a Lattice **(5)**

**Q8 a)** Let $G=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix}: a,b,c,d \in R, ad-bc \neq 0\right\}$ with matrix multiplication as the binary operation, show that G is a group. **(5)**

**b)** Let G be a group of finite order n and H is a sub group of G. Then show that the order of H divides the order of G. **(5)**